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GEOMETRY AND DYNAMICS IN THE ART OF LEONARDO DA VINCI*

Abstract

After a short biographical sketch in chapter one, chapter two explains how Leonardo contributed to a "semiotics of art". Two central concepts of his analysis of art are "geometry" and "dynamics". The application of these two notions to the frescopainting "Last Supper" and to sketches for it is the topic of chapter three. The geometrical solutions for the distribution of 13 persons are discussed in relation to the iconographic tradition and to the art of some followers of Leonardo (e.g., Dürer). Chapter four specializes on figural composition and valence patterns in a series of works by Leonardo (Epiphany, "Virgin in the rocks" and finally "St. Anne"). In the analysis of bodily configurations, gestures of grasping, reaching, and gaze-directions a semantic structure comparable to valence-patterns (case-frames, scenes) in sentences or short narratives is uncovered. In chapter five these patterns are related to the models from dynamic systems theory and social psychology. The final chapter asks if the geometrical and dynamical features discussed are also relevant in the analysis of modern art.

A first version of this paper was given at the Aarhus Conference on "Semiotics of Art" in 2001. Subsequently the topic was treated in lectures at Limoges (France) and Urbino (Italy). More specific elaborations lead to papers given in Kassel (2002) and Frankfurt (Oder) 2005. The publications Wildgen (2004, 2005 and 2006) in French and German treat more specific aspects of the topic.

1. Introduction

The term "art of Leonardo da Vinci" refers first to his famous paintings, of which few are finished and well conserved, and to the sketches and drawings which prepared his paintings and documented his scientific studies. Secondly, his theoretical thoughts on art and aesthetic s have been collected after his death by Francesco Melzi and gave rise to the "Trattato della pittura" (codex Urbinatus 1270)¹. On the basis of the theoretical positions found in Leonardo's "trattato" and in his notebooks (cf. MacCurdy, 1977) I shall try to elaborate the semiotics of art explicit and implicit in the work of Leonardo da Vinci.

Leonardo da Vinci was born in 1452 in Vinci near Florence. The big economical and political centers in Italy were Venice, Naples, Milan, Florence, and Rome. Leonardo made the first steps of his career in Florence where he worked under Verrocchio, became master in his workshop, and finally became an independent master in 1477. When in 1482 several famous painters were called to Rome in order to achieve paintings in the Sistine Chapel, Leonardo was not considered and asked Ludovico Sforza in Milan for employment. He stayed in Milan until 1499, at which point Milan was taken over by the French troupes. Leonardo left Milan via Mantua, Venice (where he stayed for a short period) and Bologna before returning to Florence. In 1506 Leonardo was invited to Milan by the French governor Charles d' Amboise and worked there until 1513. He left Milan because of the pestilence and went to Rome where he was the guest of Guilano de' Medici (1479-1516), who was the brother of Pope Leo X. Rome was in this period the center of Renaissance art, but Leonardo did not participate in the great works in the Vatican and left Rome after being invited by François I, king of France, together with his pupils Melzi and Salai. From 1516 until his death in 1519 Leonardo lived in the Castel Cloux near Amboise in France.

2. What is "Semiotics of Art" in the context of Leonardo's work?

2.1 The semiotics of painting, music, poetry, and science

Many of his original texts illustrated by sketches and drawings have been edited since; cf. MacCurdy, 1977, Clayton 1996 and Codex Leicester, 2007. All citations from the "Trattato della pittura" in the following refer to the critical edition by Pedretti, 1995.

In his "Trattato della pittura" Leonardo relates the art of painting with poetry, music, and science as an art based on mathematics (mainly geometry). If Leonardo thinks that the painter is master of all types of art, i.e., that painting is not only a science but that it has aspects of poetry and music, he presupposes a common basis for all four of the following domains: painting, poetry, music, and science. This means that he refers to a universal symbolic activity of man. It is the universality of Leonardo which makes him a semiotician, and it is his life-long reflection on the principles and on the "science" of painting ("scienza della pittura", second part of his treatise) that makes him not only a theoretician of art but also of sign-usage. In the movement of Renaissance artists from a cultural practice, learned in the workshop of painters, musicians, and poets, to a reflection on the universal principles underlying this practice and the consequent development of new practices going beyond the "maniera" of their predecessors, Leonardo establishes the stage not only for the rapid further development of painting and art, but also for the rapid evolution of science. Although Copernicus's "Commentariolus" began to circulate only after 1515, Leonardo may stand for the new generation of intellectuals in the time of Copernicus and the "modern" civilization of art and science in the 16th and 17th century, which is the basis of contemporary science.

In his treatise Leonardo states that the objective of painters is to represent mainly two things: man and his mind². The nature of man becomes visible, and therefore, accessible to the eye in the different movements, and in the proportions of his body parts (cf. Leonardo da Vinci, 1977: "Parte Terza. De vari accidenti di membra"). In order to represent man and his mind the artist must first create a pictorial space, which is the foundation for the topic of the painting. The basic technique rediscovered and further developed in Renaissance time is called the "linear perspective"; i.e., the artist must be able to represent the third dimension with the means of a pictorial plane. Secondly s/he must consider light and shadow before placing objects in space. Finally, landscape, sky, objects, animals, and persons included in the painting (mostly individuals or groups of individuals) must be arranged in space relative to light and shadow. The central concern is therefore the composition of the topic and

Cf. Pedretti, 1995: §180 (p. 219): "Il bono pittore ha a dipingere due cose principali, cioè l'omo e il concetto della mente sua. Il primo è facile, il secondo difficile, perchè s'ha a figurare con gesti e movimenti della membra;".

the choice of those postures which are able to represent the motion and the mind of the central persons.

In the context of cognitive semantics (cf. Wildgen, 2008a for an overview) we could consider the following levels of semiotic analysis:

- The space (time) of the scene depicted (perspective, light/shadow, outfit of the scenario).
- The thematic persons (the Virgin alone, together with Jesus, with Jesus and John, with Anne and Jesus). These thematic persons (or animals, e.g., a lamb) constitute a relational schema which is represented by their relative positions in space (e.g., the arrangement of apostles at the table), the static relations (e.g., the Virgin sitting on the knee of St. Anne) and the movements (e.g., the relative movement of the head in relation to the trunk, the gestures of the hands, and the direction of the gaze).³
- A narrative content related to a known episode, e.g., the moment when Jesus just said that one of the apostles will betray him in the context of the "Last Supper". The scene may be identified as one moment resulting from a series of prior events and having specific (known) consequences.

In the following I will analyze the two groups of paintings of Leonardo da Vinci in the order of these three levels.

2.2 Space, light/shadow and the outfit of the scenario in Leonardo's paintings

The mastery of perspective is an inheritance of the following artists of the 15th century: Brunelleschi, Masaccio, Ucello, Mantegna, Bramante, and Piero della Francesca. Leonardo has left no special treatise on perspective (it was probably lost), but his remarks on light and color show that he presupposed a theory of perspective similar to that handled by Alberti and Piero della Francesca. Leonardo's basic aim is not so much the scientific reconstruction of our way of seeing but an optimal rendering of "relief" by the use of light and shadow.⁴

³ Cf. for a detailed analysis of the gestures in Leonardo's work Wildgen, 2005.

^{4 &}quot;§ 124: Dico essere più difficile quella cosa ch'è constretta a un termine, che quella ch'è libera. L' ombre hanno i loro termini a certi gradi, e chi n'è ignorante, le sue cose fieno senza rilevo, il quale rilevo è la importanzia e l'anima della pittura." (Pedretti, 1995:199) His specific technique may be due to the fact that he had learned in the workshop of the sculptor and painter Verrocchio (1435-1488).

Rules in painting concern mainly the domains of restriction and not the domains of liberty; i.e., they concern the transition between a bad towards a good painting. Beauty asks for an adequate management of light and color for giving the proper significance/relief to the central topics (cf. Clark, 1958: 76 f.). The relief cuts the face against a light background (e.g., a window) with parts of the face lost in the shadows (cf. Pedretti, 1995: § 86f). In relation to early Renaissance the perspective is no more a mean to render realistically the "window of the eye"; the "relief" is rather a figure of significance and a technique which creates meaning in the picture. Leonardo's art goes beyond mimesis of nature and discovers the internal meaning of space, light and shadows for the viewer. The beauty of a scene is the recovered meaning in the mind of the viewer. Leonardo's aesthetics are nearer to Cézanne than to classicism because the creation of meaning dominates the representation (imitation) of the external world. Although in his writings Leonardo is scientific and argues mathematically using geometrical constructions, his painting is "impressionistic" in the sense given to this term by Cézanne and his followers (cf. Merleau-Ponty, 1989). However, Leonardo does not "lose the objects" and for him the superiority of painting over poetry is founded in its immediate correlation with nature, which creates a degree of trueness and security not accessible by language.⁵

The eminent role of limits and their gradual nature may be associated with catastrophe theory (a model of limits), fuzzy logics (a model of smooth boundaries), and prototype theory (profiles may be prototypes of figural perception).

By highlighting movement and accident, Leonardo asks for an interpretative activity of the viewer based on his experience. Movement and action may be extrapolated from an instantaneous picture to a process which has caused it and which will bring it to a proper end later. The interpretation of snapshots of motion is most prominent in human bodies, in the gestures of the hand, and in the postures of the head. Mimic is represented in the painting because it contributes to a language of the body which is

^{5 &}quot;§ 7: La pittura rapresenta al senso con più verità e certezza l'opere de natura, che non fanno le parole o le lettere, ma le lettere rapresentano con più verità le parole al senso, che non fa la pittura. Ma diremmo essere più mirabile quella scienzia che rapresenta l'opere de natura, che quella che rapresenta l'opere del' operatore, cioè l'opere degli omini, che sono le parole, com'è la poesia, e simili, che passano per la umana lingua." (Pedretti, 1995: 134). In the modern context of elaborated theories of vision and the invention of photography, the complex activities of Leonardo as drawing anatomist and physiologist, and as observer of the dynamics of machines and bodies are redistributed into the sub-fields like scientific photography and creative art work.

so restricted in its categorization that one instance, one moment may stand for the whole (episodic) gestalt.

2.3 Geometry and dynamics in Leonardo's semiotics of art

Under the label "geometry" I will deal with the first part called "linear perspective" by Leonardo. It concerns primarily the different shapes of objects at different distances. The painting is like a plane inserted between the eye of the viewer and a scene. Any object may be conceived as the basis of a pyramid pointing to the eye and cut by the plane of the painting. Geometry is involved also in the proportions of parts and the whole, e.g., the whole groups of the apostles (with Christ) and the subgroups they form, the whole body and its parts, such as the head, trunk, and limbs, and also the proportion of the whole room where the supper takes place, from the ceiling, to the windows, the table, etc. (as in the case of the "Last Supper"). This type of geometrical proportion may be arithmetically expressed (e.g., by fractions) and can thus be related to musical harmonies.

Dynamics in Leonardo's thinking are Aristotelian, i.e., the impetus transferred by the mover to the moved object diminishes with time and finally goes to zero. It forms another type of pyramid now pointing to the state of rest.⁷ In the Codex Atlanticus Leonardo says:

"Speak first of the movement then of the weight because it is produced by the movement, then of the force which proceeds from the weight and the movement, then of the percussion which springs from the weight of the movement and often the force."

(C.A. 155 v.b.; cf. MacCurdy, 1977, vol. I: 477)

"Force I define as an incorporeal agency, an invisible power, which by means of unforeseen external pressure is caused by the movement stored up and diffused within bodies which are withheld and turned aside from their natural uses."

(MS A, Institut de France, 34v; cf. MacCurdy, 1977, vol. I: 493)

Leonardo distinguishes: Linear Perspective, Perspective of Color and Vanishing Perspective; cf. MacCurdy, 1977, vol. II: 222 f., translated from MS 2038 Bib.Nat. 18r. The perspective has itself a dynamical interpretation as it is associated with an energy which has its seat in the eyes.

⁷ The semantics of "force" has been analyzed by Talmy (cf. Wildgen, 2008a: chapter 4) and in catastrophe theoretic semantics (cf. Wildgen, 2008b for a recent discussion)

The mathematical and physical horizon is still that of Aristotle and his medieval and Renaissance commentators⁸; only after Galilee and philosophically after Hobbes (e.g., in his "Leviathan, 1659) will these types of dynamics be overthrown. As I shall deal primarily with a kind of psychological dynamic shown in the composition of Leonardo's painting this difference will be of minor importance.

In bodily movement, any movement of one body-part has a counterpoise in another body-part. The underlying law is that of the arms of a balance or of levers in general.

"The arms of the balance make of themselves a counterpoise from the one to the other; which counterpoise will have with these arms as many varieties as the proportions of these arms will be varied." (Forster Bequest MS. II, 155v.; cf. MacCurdy, 1977, vol. I: 573)

The balance, the weight and counterpoise mean in painting that a single body is in balance if the weight of the movement of one part of the body, e.g., the head, has a counterpoise in another, e.g., in the movement of the shoulders or the trunk. The balance could be easily realized if all bodies were static. But this would make them "wooden", i.e., unanimated. The painter who wants to show the mind of the persons in the scene must show them in movement, and the balance of a person or group of persons has to be a *dynamical* balance.

The primacy of movement is grounded in the primacy of the mind. The painter should represent these actions as an expression of the mind:

"Every action must necessarily find expression in movement. To know and to will are two operations of the human mind. To discern to judge to reflect are actions of the human mind." (Codex Trivulziano, 65a; cf. MacCurly, 1977, vol. 1: 65)

The two basic pillars of Leonardo's semiotics of art are:

- Perspective (linear, of color, vanishing), which we shall deal with under the topic of *geometry*.
- Dynamics (force, weight, counterpoise, balance, movement, percussion, etc.).

In the case of Renaissance art there is a consciousness of the geometrical and even technological relevance of art (Edgerton, 1980 calls the Renaissance artist therefore a "quantifier"). Moreover, geometry is considered as a practical science, nearer to

⁸ The medieval authors are Grosseteste, Bacon, Witelo and Pecham; cf. Frosini, 1998a: chapter 7.

craftsmanship than to natural philosophy (= physics). I will ask in the last chapter if geometry and dynamics are also relevant for the understanding of modern art.

3. Geometry and dynamics in Leonardo's "Last Supper"

The general disposition is given by the rectangular table on which all thirteen persons (Christ and his twelve apostles) are placed almost in a line (two of them are sitting or standing to the right and left of the table). The total surface of the wall in the dining room is subdivided into three layers: a basic layer including a door which has been broken into the wall later, the painting of the Last Supper and three painted arcades above.

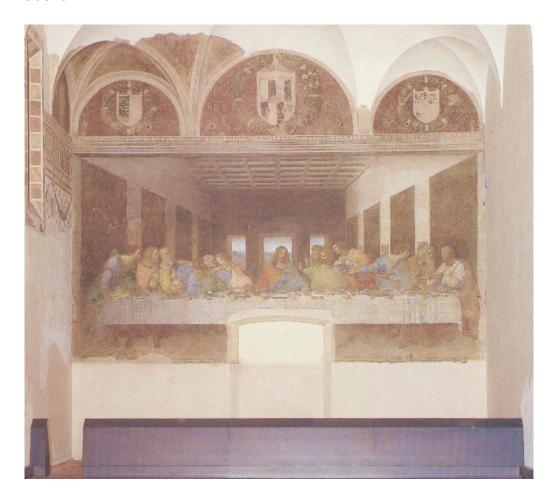


Figure 1: Wall of the refectory of St. Maria delle Grazie in Milan with Leonardo's painting (cf. Leonardo, 2001: 84).

The middle layer showing the Supper produces the illusion of a deep hall with rectangular tapestry on the sides and three openings going into a landscape. The linear perspective points to the head (the right ear) of Christ. The body of Christ with

his extended right and left arm forms the central pyramid. Jesus' ear is significantly related to the narrative content. Jesus has just uttered that one of his pupils will betray him, and he listens to their answers.⁹ His head is slightly decentered in relation to the open door in the background, which puts his face into relief against the landscape. The upper border of the landscape defines a line on which the eyes of Christ are placed.¹⁰

The geometrical arrangement of the thirteen actors in the scene has a basic symmetry: Christ versus twelve apostles, with six of them sitting to his left and six to his right. The linear arrangement, which includes Judas the traitor, is new. In most paintings that have treated the same topic before, Judas is sitting on the other side of the table and turns his back to the viewer. This is the case in Leo de Castagno's Last Supper (1447) shown in Figure 2. Even in Ghirlandajo's painting made in 1480 (cf. Wölfflin, 1904: 26 f.) and in Perugino's painting for Sant' Onofrio in Florence around 1490 (cf. Luchinat, 2001: 48) this traditional constellation is still used.



Figure 2: Leo de Castagno's Last Supper (1447).

Different sketches by Leonardo (ca. 1493/94) show that he originally aimed at a more traditional composition where Judas is sitting with his back to the viewer and John is sleeping on the table in front of Jesus. In a partial sketch Judas is standing up to grasp at the bread according to the dictum of Christ that the traitor is the one who

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⁹ Luca Pacioli, the friend of Leonardo, gave the interpretation that Christ has just said: "Unus vostrum me traditurus est" ("One of you will betray me") and that Christ in this moment accepts the beginning of his martyrdom.

In the restored painting it became clear that the eyes are at the level of the nearer landscape, i.e. on a line with the tower of a church to the left of Jesus (cf. Leonardo. The Last Supper, 2001: 157).

first takes the bread. In the final version Judas is still grasping at the bread while he holds his purse with the other hand. The final solution integrates John and Judas into the line of twelve apostles and allows four groups of three persons around Christ. John and Judas are in the same group and form an opposition inside this group.

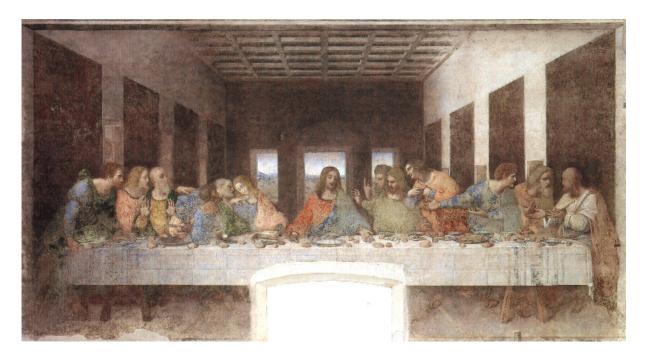


Figure 3: The whole painting in the restored shape.

Leonardo's subdivision of a group of entities on a line may be compared to a system of tones on a scale, and Leonardo has also said that since music is composed following the proportions, the same intention could be realized in a painting (cf. Braunfels-Esche, 1984: 113). The geometrical features of the painting become significant by their deviation from the normal case of realistic representation:

- 1. The viewpoint is above the heads of the real viewers in the hall (cf. Figure 1); therefore, the scene on the wall becomes similar to a scene in a theater. It is explicitly a *symbolic* representation of the Last Supper.¹¹
- 2. The table fills the whole breadth of the hall (in the painting) and leaves the room behind almost empty; i.e. the table is the scenario of the narrative content.

As a basic rule of linear perspective the central point attracting the rays of the perspective should be at a distance from the floor corresponding to that of a viewer, i.e., 150 to 180 cm (cf. also Kemp, 2001).

- 3. The persons and the associated gestures and mimics are made as large as possible. They could not sit down and eat at the table, and they have to turn their side for the viewer in order to be fully visible. Leonardo thus reduces the realism of his painting in order to maximize its expressive power.
- 4. The "semiotic" body-parts: hands and faces are dramatically emphasized by the variability of their forms.

Thus, geometry, perspective and proportion are subordinate to the expression, which is the message of the painting; the geometrical "mistakes" become iconic signs constituting the skeleton of a narrative. The dynamics of the painting are also partially narrative. The utterance of Christ "one of you will betray me" is a force, and the effect of this force creates a "percussion" in the group of apostles. Like a shock-wave it hits the two groups most strongly that are sitting at the right and left of Christ and to a lesser degree the exterior groups. If we consider the nearer groups, Jack is pushed back, whereas John, although displaced in relation to Christ, stays calm. Judas steps back just in the moment he is grasping at the bread. These two groups are more agitated than the calmer outer groups. Thus, the dynamical effect of the words of Christ is represented as a wave with repercussions and vortices (cf. Figure 5).



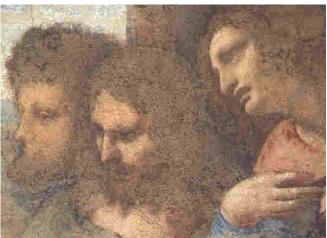


Figure 4: The two interior groups: Judas-Peter-John and Thomas-Jack-Philipp.

We can notice that the three persons subdivide again into two plus one:

John / Peter + Judas (in the center, obscured by the light coming from the left).

Thomas / Philip + Jack (in the center, receiving the full blow of Jesus' utterance).

The exterior groups have also one central figure to which the others turn. If we analyze their postures and gestures, we can further decompose all four groups of apostles into two plus one (center). The central person neutralizes and then stops the movement initiated by Jesus and thus brings it to rest. This is the natural locus of movements in Aristotle's physics. It is as if the blow of the utterance has dynamically shaped the four groups and their subgroups as a result of the "percussion"; the grouping would in this view be a natural consequence of the underlying dynamics. If a flow of water is obstructed, specific forms are created by the flow; if three objects stand in the flow, as shown on sheet 13B of Leonardo's Codex Leicester (ibidem: 90), they are bound together.

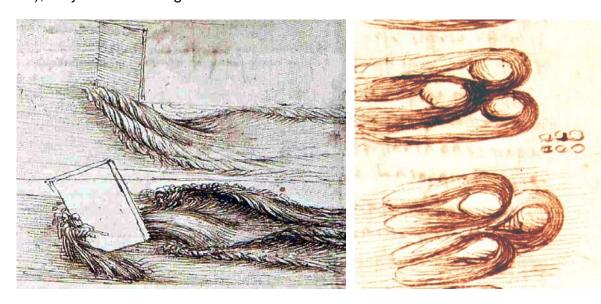


Figure 5 Details of Leonardo's treatment of hydrodynamics; c. 1509-11; cf. Codex Leicester, 2007: 47 (left) and 90 (right).

The gestures have their own language, and Leonardo asks the artist to observe people who are unable to speak in terms of how they express themselves with the help of gestures. ¹² Thus, the dynamics in the picture concern the effect of Jesus' utterance in the postures of his disciples and in their gestures and mimics. Dynamically, Judas is clearly separated from the other apostles, he seems to be lost for any positive effect, holds his money in the right hand, and stops grasping for a moment at the bread. He shows a closed, sinister face, which is in full contrast to the face of John and is illuminated by the light from the left.

¹² "§180 ... perché s' ha a figurare con gesti e movimenti delle membra; e questo è da essere imparato dalli muti, che meglio li fanno che alcun' altra sorte de omini." (Pedretti, 1995: 219)

In relation to the geometry of the painting we could say that Leonardo tries to organize his composition as an instant in a process which shows the origin of the force and the immediate effects and multiple structures created by the percussions of the force, which are in and come from Jesus. The geometry is subordinated to these dynamics. As the emotional and intellectual effects of the central force are the main topic of the painting, Leonardo reorganizes the geometry of the scene in order to arrive at an optimal representation of the percussion in body-postures, gestures and mimics.

The anchoring image schema is a kind of fluid wave, and the pattern shown in the painting is a snap-shot of the process of pattern-formation. The word of Jesus has the effect of a "Turing instability", which generates morphology by self-organization in the group of apostles; the four triads of persons are the product of the interaction of the expanding instability in the subgroups. The whole is then a type of self-organized pattern resulting from the instability created in the center (Jesus). Leonardo's construction is a visualized solution under these premises. This explains why Leonardo was perceived as a radical innovator in his time. His solution was partially taken up by Raphael and Dürer in their representations of the "Last Supper", but the dynamics were less radical. They chose a compromise between the lesson given by Leonardo's Last Supper and tradition.

The shape of the table is a basic factor for the grouping of the apostles. In Dürer's woodcut (1510) the table is less broad and the apostles form a circle with an opening on the front-side. The woodcut is leonardesque in the postures and gestures of the apostles, but its geometry follows another tradition. Thus Dirk Bouts' "Last Supper" (1467, town hall, Leuven) shows the apostles sitting along a quadratic table, with two

In Frosini (1998a:138-148) it is shown that the Aristotelian idea (De caelo, II,6) that the wave of water accelerates until it reaches the mid-line of the path towards the limit and decelerates later has been commented on and discussed in medieval time and during the Renaissance. This could explain why the two groups near to Jesus at his left and right show a stronger movement. Leonardo also considered backward waves called "retrosi" and "vortici" (cf. the comparison of Leonardo's sketch of the flow behind a cylinder and modern representations in: Gombrich, 1969: 191). The motion of Peter and Philip towards Jesus in the nearer groups could be explained by the vortexeffect of the wave behind an obstacle. In the same period, the theory of waves (in water and air) became de-mechanized (cf. Frosini, 1998a: 144) and could be applied in semiotic contexts.

¹⁴ The engraving by Marc Anton after a lost painting of Raphael takes up the theme of the silent Jesus in a frontal perspective and the apostles grouped by their reaction to the sentence just spoken by Jesus. The group is sitting against an architecturally spaced wall with two apostles on the sides of the table; this transforms Leonardo's linear array into a rectangular one.

sitting with their back to the viewer, three on each side of the table, and four at the side of Jesus.¹⁵



Figure 6: Dürer's "Last Supper" (1510). Woodcut from the series of the Great Passion (cf. Hamann, 1932: 500).

The general distribution of elements on the surface of the painting follows basic laws of gestalt described in Gestalt-psychology (cf. Verstegen, 2005). The corners and sidelines establish a formal force-field:

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337).

An almost circular arrangement of the apostles is found in Dürer's woodcut in the series of his "small passion" (1509-1511). Contrary to Leonardo he conserves the specific position of John in the arms of Jesus in all his woodcuts and drawings. In a drawing of 1523 Dürer returns to the classical (Italian) oblong table and he departs radically from the younger tradition by placing Jesus (and John) to the left of the table (cf. Panofsky, 1977: 295; plates 277f.; and Marani, 2001: 334-

"The energetic force emanating from opposite corners toward the center reinforced in pairs, gives rise to potential diagonal lines or vectors which connect opposite corners."

(Saint Martin, 1990: 97)

In Leonardo's "Last Supper" the regular and symmetric construction of the surfaces is prominent. The upper rim of the tapestry heavily reinforces the diagonal system. One can complete these diagonals and obtain a rectangle whose baseline is on the level of the feet of the apostles (visible under the table).

The X formed by the diagonal is the first basic feature. The second is given by a cruciform structure also centered on the head of Jesus. The horizontal line of this cross goes through the heads of the apostles.

As Saint Martin (1990: ibid.) points out, this "second armature of the Basic Plane" (ibid.) indicates "a decreasing and influent movement of the energies of the four angles toward the central regions of the plane" (ibid.). Thus, the horizontal (middle line) furnishes a dynamical background with forces coming from the periphery and annulled in the center. The "percussion" of Jesus' word, i.e., the shock-wave going from the center horizontally to the left and right border, is thus competing with the general force-field immanent in the rectangular plane.

The geometry of a composition in painting and the arithmetical ordering of figures depend on three presuppositions:

- the shape of the painting, i.e., rectangular (horizontally, vertically) quadratic, circular; this is the *geometric* frame,
- the linear *perspective*, which produces the illusion of depth,
- the shape of the *architecture* (e.g., the walls, a stair) or of *furniture*. In the case of the "Last Supper", the shape and relative size of the table is the crucial factor which governs the geometrical organization of the persons sitting at or standing along the table. ¹⁶

In the medieval and orthodox tradition the quasi-mathematical perspective used by Renaissance painters was not known, and different principles of representing and viewing were used. Thus in order to optimally distribute persons sitting at a table the front line was straight, whereas the three other borders were disposed along a semi-circular line; thus, depth and the linear ordering of the main topics in the picture were both achieved. In cognitive terms one could say that the scanning of the picture was not unified. Important elements like persons represented were dominant against secondary features like tables and other furniture. In a group of persons the central person was highlighted, less important persons were treated on another level. The thematic hierarchy was more important than the overall perspective subordinated to one vanishing point. Semiotically, such

As to the arithmetic order, there is a principle of chunking which operates. Although in psychology the "magical" number was considered to be 7 ± 2 (cf. Miller, 1956), we observe that triads and dyads are of major importance. Possibly, Peirce's semiotic hierarchy which considers the triad as the basic configuration is also valid for the grouping of persons in paintings. The "magical" number seven can be decomposed into triads (3+3+1) or dyads (2+2+2+1). As the number of apostles goes beyond the number nine (seven plus two), it has to be separated into several groups, e.g., symmetrically (6 + 6). All groups of persons bigger than 9 and not symmetrically grouped may be considered as "crowds". In Leonardo's "Epiphany" such a crowd surrounds the central scene (the Holy Family and the Magi). In the representations of battles, e.g., in Albrecht Altdorfer's "The battle of Alexander" (1529), the innumerable soldiers appear like a texture of the landscape. In his "Battle of Anghiari" Leonardo concentrates the battle theme around a group of four horses with four horsemen competing for the standard. Rubens, who in 1603 made a copy of Leonardo's painting, used such a composition again in his painting "The abduction of the daughters of Leukippos" (around 1617). Here, two horsemen kidnap two women. 17

4. The semiotics of a figural composition

Christ and the Virgin are central topics of Christian paintings (like Hercules and Venus are in the context of antique and neo-antique paintings). The "Last Supper" shows Christ in the group of apostles. The Virgin appears very often with Jesus as a child (in different postures and functions). Sometimes both are surrounded symmetrically by two saints or by two angels. These configurations are basically dyadic; i.e., the central pair, the Virgin and Jesus, are the main topic (the figure), and the surrounding persons are part of the background. In the case of the Holy Family, St. Joseph is added to the group of the Virgin and Jesus, but although he is

an order seems to be more natural than the geometrical order of mathematical perspective. Cf. Antonova and Kemp (2005: 421-426) on scanning and "reversed perspective".

Rubens composed his last supper (Alte Pinakothek, Munich) in the tradition of Flemish painters. The group of apostles surround a quadratic table with an opening in the foreground. On the right edge of the (almost) circular group of persons sits Judas turning into the direction of the viewer (and the door). The degree of motion and the gestures and mimic dynamics are leonardesque, but they lack the clear geometrical and dynamical order in Leonardo's construction. This evolution seems to be inherent to the development of late Renaissance and Baroque art. Italian artists like Jacopo Bassano (1510 - c.1592) in the sixteenth century even increased the degree of motion, expression and psychological differentiation (cf. Bassano's Ultima Cena, 1546-1548, shown in: Marani, 2001: 310f. and commented in: Wildgen, 2004).

biographically associated with both (but not the father of Jesus), he has rather the status of a bystander ("circonstant" in Tesnière's' terminology). In Peirce's terminology we could say that the "Holy family" is a degenerated triad. ¹⁸ In the first painting of Leonardo as independent master (ca. 1481/2) the visit of the three Magi, the Magi kneel in front of the Virgin and Jesus in a triangular construction, and they keep a distance which marks their secondary role (in relation to Mary and Jesus). A half-circle of about twenty persons separates them from a background which has no direct relation to the episode. ¹⁹

Sketches for the painting show that geometrical considerations were basic for its composition. Thus, the linear perspective is defined by the architecture in the background. The vanishing point is on the head of a rider of a horse which rears; i.e., Leonardo takes a short instant of a non-topical event as focus of the linear perspective (cf. Figure 7).

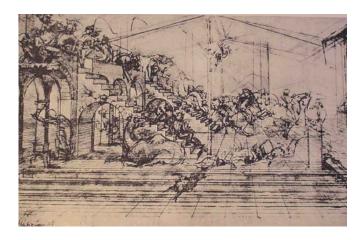


Figure 7: A sketch for the Epiphany.

In the painting, this linear perspective becomes more or less irrelevant; it only structures the fourth level of representation.²⁰ The three dominant levels follow the geometrical figures of a broad triangle pointing to the head of the Virgin and including the three Magi. The triangle is dynamically asymmetric, as the right side is the line between the gaze of Mary and Jesus that points to the face of a kneeling Magus.

Geometry and Dynamics Leonard onew. doc

¹⁸ Cf. Peirce, MS 308 (1903) in: Peirce, 1986, vol. 1: 432. If one member of a dyad is "complicated" to a couple or triple with identical function as the singular term, the whole remains a kind of dyad. Thus, if Paris is judging the beauty of three women, the basic configuration is still dyadic.

¹⁹ Some authors have counted 66 persons and animals in the painting.

Probably, Leonardo did not finish this painting (on which he had worked on for seven months, cf. Clark, 1958: 42) because too many problems of construction had been posed.

This may be called the central force-line inscribed into the triangle. The head of Mary is at the same time the vertex of a semicircle along which the surrounding figures are arranged.

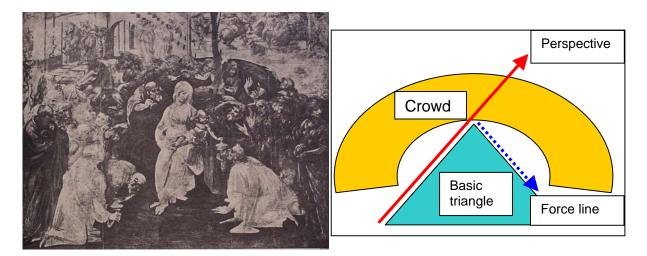


Figure 8: Leonardo's Epiphany (Adoration of the Magi) and the underlying geometry and view-lines.

In his further development Leonardo reduces the linear perspective and concentrates on the features he calls "perspective of color and vanishing perspective". The background becomes idealized, vague, and neutral (cf. his "Mona Lisa").

In "The Virgin of the rocks" we find a new constellation with four persons.²¹ The geometrical scheme is that of a pyramid with the angel on the second (visible) side of it. The front side has on its vertex the face of the Virgin, on its edges the Jesus-baby at the right and St. John Baptist as a baby on the left (above the base-line of Jesus and the angel).

In the London version of the painting, which is a modified copy, the pointing gesture of the Angel is eliminated. This and the different gaze direction of the angel simplify the quaternary construction to a triadic one.





Figure 9: "The Virgin of the rocks" by Leonardo (first version; 1492-94) and detail of the second version

This painting shows the complexities Leonardo tried to manage. Jesus is separated from Mary, who holds St. John with her right hand. In the field of Jesus' head we find three different hand-gestures:

- Mary protects/grasps Jesus (she has no contact with him and her gesture is interrupted by the gesture of the angel),
- Jesus blesses St. John,
- the angel, who looks at the spectator/interpreter outside the painting, points to
 St. John, and
- St. John looks at Jesus and kneels in front of him.

The hand gestures alone define five different force-lines, and all four persons have different gaze-lines involving different angles of the head. All these forces imply some

narrative context; i.e., Leonardo tries to compress a story in the static configuration of a painting. The basic episodes are:

- The angel Gabriel announces the birth of John (Luc. 1,13)²²
- The angel Gabriel announces the birth of Jesus (Luc. 1,35)
- John announces the activity of Jesus (Mat. 3,11; Luc 3,16; Joh. 1,26)
- Jesus calls John the prophet, who prepares his paths (Mat. 11,10; Luc. 7,27).

The biblical report is represented in its different episodes; nevertheless, the quintet Elizabeth (mother of John) - Mary / John - Jesus / and the angel Gabriel is simplified to a quartet. As the relational link between John and Elizabeth is lost, John is now linked to Mary,who in the biblical story has only a light link to him. At her arrival John moves in the womb of Elizabeth. ²³

Geometrically, the four persons who interact occupy the edges of the pyramid of persons; the gestures of their hands and their gaze-lines constitute a "pyramid" of forces, as shown in **Figure 10**. One could say that this painting lacks a proper center of gravity. Its thematic units are distributed in space, and although there is a geometrical schema for this distribution, the dynamics of weights, forces, and gestures remain somewhat vague. In a second version (in London) the intriguing gesture of the angel and its vision-line out of the frame are eliminated. This creates a better balance, but diminishes the vividness.

²² Some commentators call the angel "Uriel". If we compare the two angels in the present painting and Leonardo's "Annunciation", they have similar clothes and colors. Gabriel is in general associated with Mary and with John. Piccaluga (1998) also calls the angel in Leonardo's painting "Gabriel".

Gabriella Piccaluga (1994) relates Leonardo's painting to the teaching of Amadeo Mendes da Silva in Milan. He had lived in the Franciscan cloister San Francesco Grande for which Leonardo painted "The Virgin of the rocks" (around 1494). Mary (and with her John Baptist) symbolizes the figure of Sophia (in the gnostic tradition). The strong link of the group Mary - St. John Baptist and their contraposition to the Angel Gabriel could be an interpretation of the teaching of Mendes which was later rejected as heretical. In Mendes' teaching, the angel introduces the third immaculate being, Jesus, to the first two, Mary and John Baptist. Together the Virgin, John and Jesus form a trinity in the theological interpretation of Mendes.

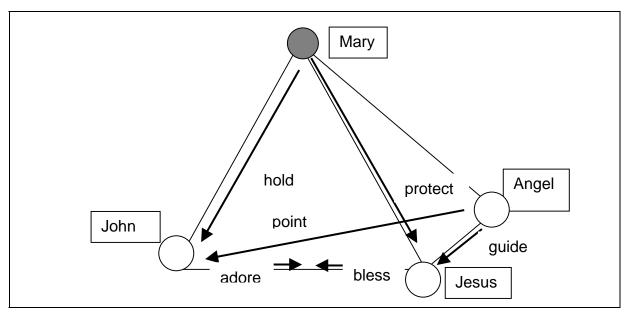


Figure 10: The pyramid in the "Virgin in the Rocks".

The elaborated cartoon for St. Anne (with Mary and Jesus) was finished in 1498/99, i.e., after the "Last Supper" and a decade after "The Virgin of the rocks". It shows another quaternary relation: St. Anne, Mary, Jesus and St. John. The major narrative and topical contents are:

- St. Anne is the mother of Mary,
- In the tradition of this pictorial topic, Mary is sitting on the knees of St. Anne and Jesus is held by Mary.
- The presence of John or the lamb is facultative.

In the elaborated cartoon (now in London), Leonardo cuts the upper edge of the pyramid such that the heads of Mary and Anne are on one horizontal line. Nevertheless, the contours of Mary at the left and the gaze-line of Mary—Jesus—St. John form a triangle. Dynamically, we have a central triad given by Mary (holding Jesus), Jesus (blessing St. John) and St. John (receiving the blessing); St. Anne is a bystander. She supports Mary (on her knees), looks at her and points to heaven; her pointing hand defines the upper edge of a smaller pyramid, with the head of Jesus and St. John as the bottom-line. **Figure 11** illustrates this analysis.

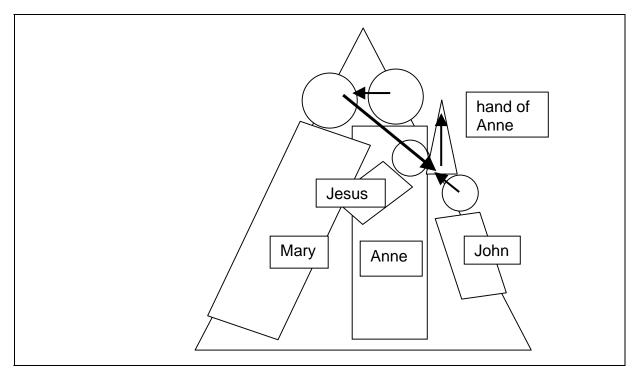


Figure 11: Geometrical figures and force-lines in the London cartoon of St. Anne.

The lines of the gazes and hand gestures fit into a rectangle; its diagonal is the basic force-line, which has Jesus as its central attractor. The weights of the central groups of adults Anne/Mary are balanced by the body of Jesus, which is a counterpoise to the body of Mary sitting on Anne's knees. The whole composition is, therefore, centered on Jesus (as its "barycenter").

In 1503 and later, Leonardo tried different constructions. In one sketch, the head of St. Anne in proximity to that of Mary is scratched, and a new version where the heads of Mary and St. Anne are at a certain distance, so that St. Anne does not look at Mary, is given (cf. Clayton, 1992: 245, fig. 22). The basic geometrical figure is now a triangle, with the base line on top as shown in **Figure 12**.

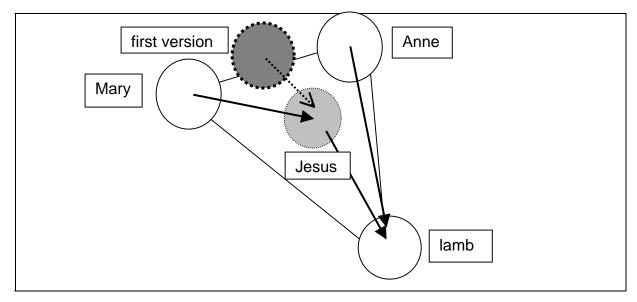


Figure 12: Triangular construction in the sketch of 1503.

The final stage (ca. 1510) of St. Anne (now exhibited in the Louvre, Paris) comes back to the asymmetric pyramid with the main force-line at the right, placing St. Anne, Mary, Jesus, and the lamb on one gaze-line. However, the dynamics are new. Although Mary is still sitting on the knees of St. Anne, she is moving towards the child (Jesus). This complicated decentralization clears the space for a full portrait of St. Anne, who now joins the main gaze-line instead of breaking it into two gaze-lines as in the early cartoon. Jesus is also in motion, and while climbing the lamb, which is struggling against it, he is held back by Mary. The dynamical scale of events goes from

- St. Anne (passive, just observing),
- the lamb (just reacting, struggling),
- Mary (gently withdrawing Jesus from the lamb), to
- Jesus (trying to ride on the lamb and moving against the force-line of Mary).

The force-lines of gaze and gesture are reinforced by (real) forces like:

- support (Anne, Mary)
- hold back (Mary, Jesus)
- mount (Jesus, lamb)
- separate from (Jesus, Mary)

The dynamics of support serve as a background for the dynamics of holding, separating, and mounting. Jesus is the mediating force between Mary and the lamb.



Figure 13: The final version of St. Anne completed in 1509/10 (Louvre, Paris).

The strong dynamism of this painting and the stricter geometrical construction are obvious, if we compare it with Raphael's treatment of a similar topic such as the "Holy Family" (1507). Although an influence by a lost cartoon of Leonardo is visible (e.g., the pyramidal construction and the gaze-lines on the left edge of it), it is more static (see Joseph's bar in the center and the linear array of the figures without weights and counterpoise).

5. Figural composition and levels of interpretation

In the last sections, I have mainly analyzed what can be seen in the painting in terms of geometrical and arithmetical structures, force fields defined by obvious movement (frozen in the picture), and of gestures and directions of glance. These visible features (I did not consider color, shades, transient zones, the landscape or the floor on which the figures stand) establish a basic signification of the piece of art, independent of the literal or metaphorical/ironical/... meaning of the objects, animals, and persons represented, or of the narrative context of the actions frozen in the picture. One could go further and study the details of perceptual dynamics, as Arnheim did in his work (cf. Verstegen, 2005: 29-35), or analyze extremes of curvatures and their implicit force-lines (cf. Leyton 2006).

In the last section on the paintings with St. Anne I have introduced the names of Anne, Mary (the Virgin), Jesus, John (Baptist), and the lamb, and I have renamed several recognizable actions, such as *support*, *hold*, *mount*, and *separate*. These referential meanings relate what is visible in the painting to the knowledge of the viewer (ideally to the knowledge of a 16th century viewer at court or in the clergy).

This level of interpretation, which links visible features to content, goes beyond the painting; i.e., we have to consider another semiotic stratum, on which persons, animals, their attributes, actions, episodes, and stories are organized. If we analyze a picture of ourselves taken some years ago, this level of knowledge could be an autobiographic memory and it is not necessarily linguistic knowledge. In the case of sacred art like Leonardo's "Last Supper" or "St. Anne", the bible is the narrative background; i.e., we refer to some linguistically organized and retrievable knowledge. As a consequence, the interpretation of the painting at the content level has to operate on two separate "mental maps" in the mind of the viewer:

- (a) The visual features (including the geometrical and dynamical features discussed in the last sections) and the entities they constitute
- (b) The relations between these entities (recoverable visually) and of forces, events and actions frozen in the painting
- (c) The meaning of entities recognizable in the painting

Visually recoverable relations (b) are:

- 1. Anne background to Mary background to Jesus
- 2. Anne's head above Mary's head above Jesus' head
- 3. Anne looks older than Mary looks older than Jesus
- 4. Anne supports Mary supports Jesus
- 5. Anne and Mary form one group, Jesus and the lamb another group (by proximity).

The interpretation of immediately given data in search for a richer understanding is called "attribution" in social psychology, e.g. the attribution of causes and motivation for a perceived event or action. In Figure 13, the three persons Anne, Mary, and Jesus (of different age and sex) are easily identifiable as the major topics. From our knowledge of the bible we know that there is a kinship relation: Anne — mother of — Mary —mother of — Jesus (by transitivity we know that Anne is the grandmother of Jesus, by inversion that Jesus is the son of Mary and Mary the daughter of Anne). Even the colors may help to match the knowledge map with the visual map. Traditionally, Anne has a green mantle and a red suit. In Leonardo's painting mother and daughter have a green mantle, and Mary wears a red suit. This knowledge level constitutes a space of conventionalized meanings related to the visual space that we establish by looking at the painting (a) and inferring in a kind of imagined mirroraction²⁴ the relevant forces (b).

The knowledge space is, however, much richer:

- The lamb is a symbol of the sacrifice of Jesus and an attribute of John (Baptist).
- From John, who replaces the lamb in the cartoon, we may remember the specific relation of John to Jesus; John baptized him and called him the Messiah.
- Further links exist between John's mother, Elisabeth, and Anne, the mother of Mary; both had been waiting long to become pregnant. Further, Elisabeth's pregnancy announced Mary's, and Mary stayed with Elisabeth during the first months of her own pregnancy.

I will not further elaborate the contents of the attributional space and its consequences for the final interpretation. My concern is rather the background of (archetypal) knowledge which governs the interpretation process (the component called "generic space" by Fauconnier and Turner, 2002).

The first space (the painting and its visual effect) contains a rich geometric and dynamical structure (weights, barycenters, force-lines, gaze-directions, etc.) which is used in many of Leonardo's paintings (and in those of his contemporaries). This is relevant in general for the selective and emergent processes in conceptual integration (cf. Oakley, forthcoming 2009). A purely static generic map would be

Mirror cells in higher primates allow the observer to reproduce in his mind the movement or action he is witnessing. In the case of a static image, he/she may reproduce the process which has led to the static situation.

insufficient for both the pictorial and the narrative (knowledge) map, and an adequate anchoring of the analysis of verbal and visual signs asks for a dynamic map. The art of Leonardo shows clearly that at the generic level we need a concept of *dynamic valence*. Peirce was the first to propose a general scheme on this level of abstraction. His monads, dyads and triads are considered as dynamical wholes not reducible by simple composition.

In the case of St. Anne we have on the surface a quaternary constellation: Anne — Mary — Jesus — lamb. If one considers the force fields and actions, one notes that a basic interaction links mainly Mary — Jesus —the lamb.

- Mary draws on Jesus.
- Jesus draws on the lamb.
- The lamb resists.

There is a conflict between Mary, who tries to prevent Jesus from mounting the lamb, and Jesus, who notices this (he looks back to her) but resists her. This triad constitutes a force field which dominates the message of the painting. A first schematic representation introduces two vector-fields with attractors:

Jesus is in the metastable position between two attractors; the narrative (biblical) content of these attractors is:

Mary: His mother; she cares for her baby.

Jesus: He feels the duty to sacrifice and to leave Mary behind.

The generic space with dynamic features can be derived from catastrophe theory (cf. Wildgen, 1982, 1994). One basic dynamical pattern, the semantic archetype of *transfer*, is shown in Figure 14.

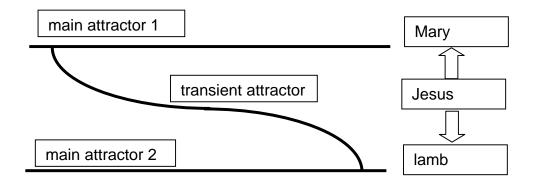


Figure 14: The dynamical archetype of transfer (giving) and a fiber on it (with attributed contents).

This archetype does not describe completely the basic interactions in the composition. We have to consider two complications:

- a) Anne supports/anchors the whole event (physically and genealogically), and she is a fourth attractor, who does not directly intervene but sustains the event (which is happening on her knees). In catastrophe theory, a second internal variable (and dimension) is introduced, and Thom (1983: 205) calls the basic archetype with four centers of attraction a "Messenger". It works like a background cause, which is a medium (cf. Wildgen, 1982: 88-92; 1994: 129-134) which enables the interaction and transfer implying three attractors and their force-fields.
- b) There is a complication in the manner of "transfer" which corresponds to the difference between the following sentences:
 - i. Mary sits Jesus on the lamb,
 - ii. Mary withdraws Jesus from the lamb,
 - iii. Mary prevents Jesus from riding the lamb,
 - iv. Mary tries to prevent Jesus from riding the lamb.

In the visual structure we just observe the hands of Mary seizing Jesus and the hands (and feet) of Jesus seizing the lamb, and we see that Jesus has a stronger grip on the lamb than Mary has on him. The turning of his head creates an opposition to the force-direction of Mary's hands.

The preference scale of the sentences: iv > iii > i is linked in the context of the painting to visual cues, but we need further knowledge which comes from our experience with human interaction. In a phenomenological perspective, the bodycentered and enacted schemata are fundamental.

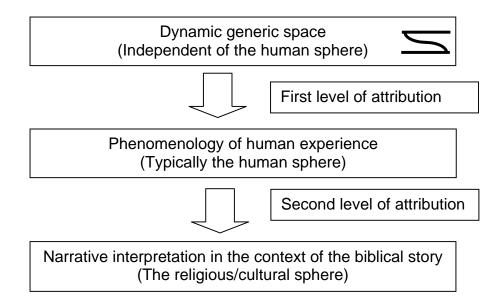


Figure 15: The generic schema and two further levels of attribution.

If one observes a mother interacting with her child, the interpretation at the second level occurs. In cultural traditions that have elaborated a collective visual or linguistic memory (e.g., of major biblical contents), the more artificial third level is superimposed on the quick routines of level two. It is typical for art that cultural knowledge and special "reading" skills are presupposed; just "perceiving" and recognizing is not enough.²⁵

6. Are geometrical and dynamic features relevant in the analysis of modern (abstract) art?

Modern art gave up the realism implied by Leonardo's theory of art and abandoned narrative contents and recognizable representations. Did geometry and dynamics loose their relevance in this context?

It is true that spatial illusion, the major effect of the correct use of linear perspective, either became secondary or was replaced as artists put the accent on the two-dimensional nature of paintings, the relevance of texture and the balance of colored

²⁵ This became obvious when in modern art the narrative content and the reference to established canons of content were abolished. Compare Wildgen (forthcoming 2009) for the analysis of order and chaos in Pollock's "Action Painting".

surfaces, characteristically in the work of Cézanne (1839-1906) and van Gogh (1853-1890). As a consequence the geometry of the plane, and the dynamics of strokes and lines came to the foreground. The trend towards abstraction and minimalism enforced the geometrical and dynamic features; in a sense it eliminated many attributional and representational processes dependent on specific cultural presuppositions. In the synthetic cubism of Picasso (1881-1973) new artificial perspectives, and modes of integration of parts to wholes, appeared. The parts, such as of a body or plant, were still representational, but the composition was artificial in relation to everyday experience. When Kandinsky (1866-1944) devised his first abstract paintings, or when Pollock (1912-1956) created his action paintings, geometrical order seemed to disappear, and stochastic (noisy) patterns and irregularity seemed to dominate.

Nevertheless, geometry did not disappear. Since the mid 19th century new models of geometry (non-Euclidean and hybrid geometries) and new types of dynamics (non-linear and chaotic dynamics) have been formally described. In relation to these geometries, Picasso's cubistic paintings and Pollock's dripping actions correspond to geometrical and dynamic principles in a similar way to Leonardo's paintings; i.e., the link between mathematics and art is still existent and productive.

It is clear that the higher attributional levels were not just cancelled; they were replaced by new, and often very complicated attributions, referring to specific cultural experiences in a global and rapidly changing world. Most of modern art criticism tries to analyze these often very unstable and unpredictable processes. This has the consequence that such analyses cannot cope with the standards of modern experimental and mathematically structured sciences. The underpinning of the attributional processes by geometrical and dynamical principles remains to be the backbone of our understanding of visual art because they are cognitively founded in our sensorial and motor-capacities. This level is accessible for scientific methods and makes a scientific visual semiotics feasible. Via the evolution of these capacities it is even rooted in the physics of the surrounding world, and therefore realistic, but in a less immediate sense than in Renaissance art.

As a consequence the cognitive semiotics of art should follow a double strategy:

- Find the universal underlying mechanism of visual understanding.
- Describe the infinity of attributional processes dependent in cultural traditions and context of usage.

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